GDOP in Multi-Static Radar

Abstract

I graduated from the University of Aberdeen with a degree in electronics and electrical engineering. As part of the finial year of my degree I completed an undergraduate thesis where I considered the concept of a multi static passive radar system capable of tracking commercial civil aircraft using transmitters of opportunity. In this entry I aim to identify the errors associated with a multi-static radar system's geometry, where they originate and how to separate them from other system errors to create a model.

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Acronyms

SESAR	Single European Sky ATM Research
RADAR	Radio Detection and Ranging
TDOA	Time Difference of Arrival
2D	Two Dimensions
3D	Three Dimensions
FPGA	Field Programmable Gate Array
GDOP	Geometric Dilution of Precession
PDOP	Position Dilution of Precession
VDOP	Vertical Dilution of Precession
HDOP	Horizontal Dilution of Precession

Introduction

As part of my undergraduate degree at the University of Aberdeen I completed an undergraduate thesis. In my thesis I considered the concept of a multi static passive radar system capable of tracking commercial civil aircraft using transmitters of opportunity.

An error is the can be defined as the difference between computed, estimated or measured value and the true, specified, or theoretically correct value. Any measurement made or position estimated by a radar system there is an associated error. Here I aim to identify the errors associated with a multi-static radar system's geometry, where they originate and how to separate them from other system errors to create a model.

Geometric Dilution of Precision

The geometric dilution of precision in a multi-static TDOA radar system is comparable to the GPS GDOP metric as described in the paper *GPS GDOP metric* (R. Yarlagadda, 2000). Adapting the metric as described in *GPS GDOP metric* (R. Yarlagadda 2000), a metric can be obtained representing the geometric error of the system separately from the error in the TDOA measurements. The configuration of transmitters and receivers relative to the target affects the precision with which the target can be located. The geometric error relates to the difference between the direct path between the transmitter and receiver and the reflected path from the transmitter to the target to the receiver. The closer the reflected path is to the direct path, the greater the error in the position. The geometric error relating to a specific transmitter i and specific axis ζ, can be calculated as in Equation 1

Equation 1

$$\alpha_{i\zeta} = \frac{\zeta_r - \zeta_{ti}}{\left|\overrightarrow{TxS}\right| + \left|\overrightarrow{SRx}\right|}, \zeta = x, y, z\,.$$

The geometric error $\alpha_{i\zeta}$ described in Equation 1 has a maximum value of one when the target lies directly on the normal between the transmitter and receiver.

Equation 2

$$\sigma_{i\zeta} = \alpha_{i\zeta} \times \sigma_{TDOA}$$

Equation 2 shows the relationship between the error position $\sigma_{i\zeta}$, $\alpha_{i\zeta}$ geometric error and the error in TDOA measurement σ_{TDOA}, where ζ is the axis and i is the transmitter. Using Equation 1 and the calculus defined in *GPS GDOP metric* (R. Yarlagadda 2000), a GDOP metric can be defined for a multi-static radar system.

Visibility Matrix

Equation 3

$$H = \begin{bmatrix} \alpha_{1x} & \alpha_{1y} & \alpha_{1z} & 1 \\ \alpha_{2x} & \alpha_{2y} & \alpha_{2z} & 1 \\ \alpha_{3x} & \alpha_{3y} & \alpha_{3z} & 1 \\ \alpha_{4x} & \alpha_{4y} & \alpha_{4z} & 1 \end{bmatrix}$$

Equation 3 represents the 3D visibility matrix for a multi-static radar system consisting of four transmitters and a single receiver. The dilution of precision parameters D_{mn}, can be obtained as shown in Equation 4.

Equation 4

$$\left(H * H^T\right)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}$$

DOP Equations

From the dilution of precision parameters, the typical DOP figures can be obtained as described in Equation 5, Equation 6 and Equation 7.

Equation 5

$$PDOP = \sqrt{D_{11} + D_{22} + D_{33}}$$

Equation 6

$$HDOP = \sqrt{D_{11} + D_{22}}$$

Equation 7

$$VDOP = \sqrt{D_{33}}$$

Using Equation 1 a 2D visibility matrix (Equation 8) can be generated for a multi-static system consisting of three transmitters and a receiver.

Equation 8

$$H = \begin{bmatrix} \alpha_{1x} & \alpha_{1y} & 1 \\ \alpha_{2x} & \alpha_{2y} & 1 \\ \alpha_{3x} & \alpha_{3y} & 1 \end{bmatrix}$$

The dilution of precision parameters D_{mn}, can be obtained as shown in Equation 9.

Equation 9

$$\left(H * H^T\right)^{-1} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

From the dilution of precision parameters, the typical DOP figures can be obtained as described in Equation 10.

Equation 10

$$HDOP = \sqrt{D_{11} + D_{22}}$$

GDOP Model

Developing a model for the GDOP error for a system the distribution can be mapped relative to the transmitters and receivers. This would be partially useful for ensuring that an approach to an airport gets the best possible level of service and that there are no GDOP chimneys where the error is effectively infinite.

Using the visibility matrix as defied in Equation 4 a map can be generated representing the GDOP error for a system consisting of three transmitters and a single receiver. Each point on the map represents the GDOP error at that point due to the relative positions of the transmitters and receiver.

Diagram Representing GDOP

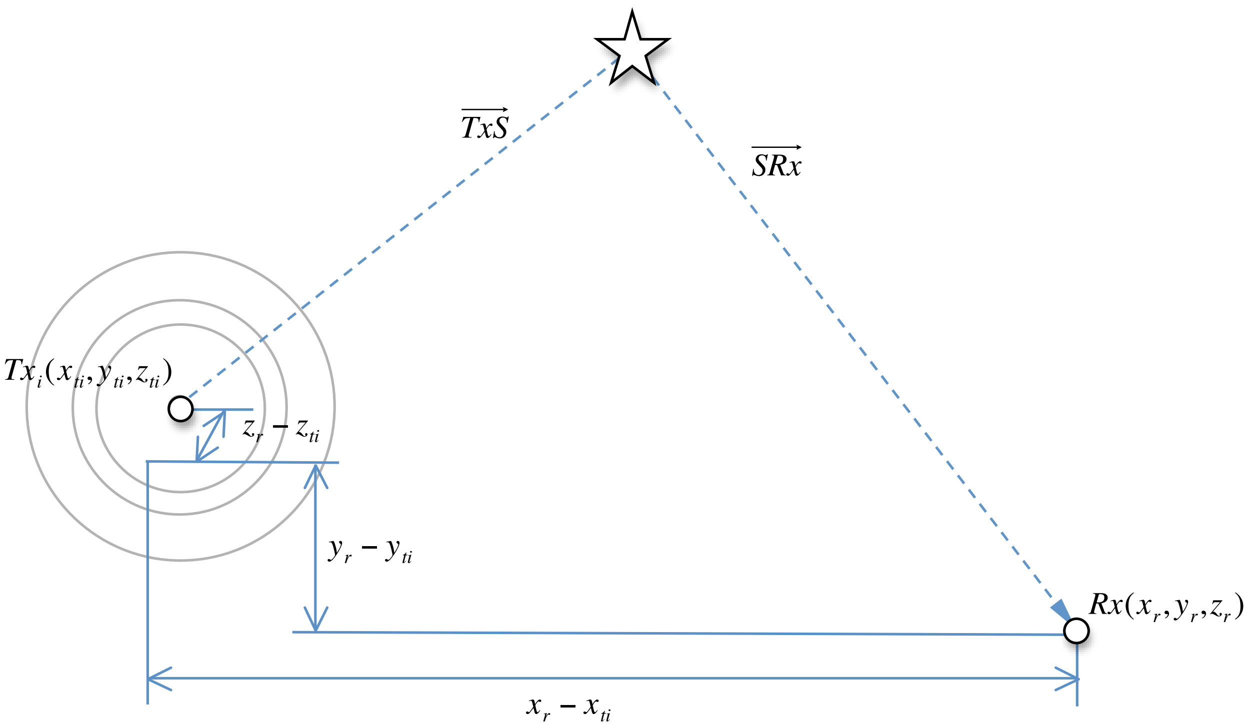

Figure 1

Figure 1 shows a representation of a single target, a single transmitter and a receiver in 3D. It can be seen here that for a fixed error in the TDOA measurement for any target position, the greatest position error occurs when difference between the multi-path signal and direct-path signal is at it's lowest. The lowest difference occurs when the target is directly between the transmitter and receiver.

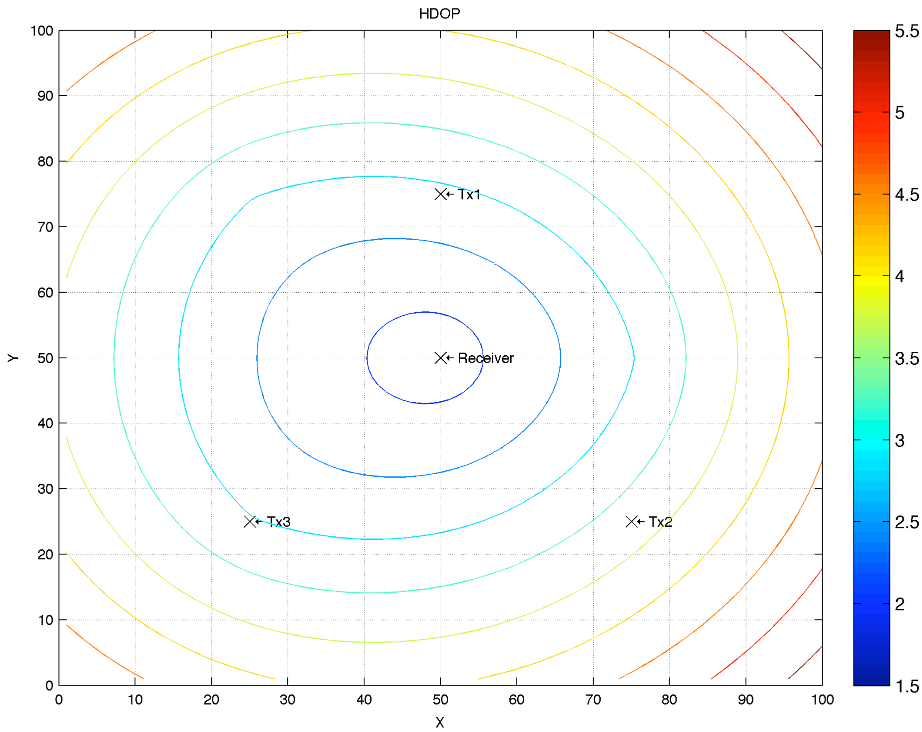

Figure 2

Figure 2 is a plot of the HDOP of a 2D multi-static radar system consisting of three transmitters and a single receiver. The higher HDOP values are red and the lower are in blue. Higher HDOP metric values correspond to higher geometric dilution of precision.

Contact Me

I would appreciate any feedback on my work, positive or negative. I would be especially interested to hear for people in industry or academia as I am currently looking for an opportunity in engineering. I am particularly interested in digital signal processing, FPGAs, algorithm design, MATLAB and system design. By far the easiest way to contact me is by e-mail andrew@chiprate.co.uk. A PGP public key for this address can be found at www.chiprate.co.uk in the contact me section.