Geometry and Operation

Abstract

I graduated from the University of Aberdeen with a degree in electronics and electrical engineering. As part of the finial year of my degree I completed an undergraduate thesis where I considered the concept of a multi static passive radar system capable of tracking commercial civil aircraft using transmitters of opportunity. This entry will describe the geometry of multi-static passive radar and how it obtains a target solution.

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Acronyms

SESAR	Single European Sky ATM Research
RADAR	Radio Detection and Ranging
TDOA	Time Difference of Arrival
2D	Two Dimensions
3D	Three Dimensions

Introduction

As part of the finial year of my degree at the University of Aberdeen I completed an undergraduate thesis. In my thesis I considered the concept of a multi static passive radar system capable of tracking civil aircraft using transmitters of opportunity.

This entry will describe the geometry of multi-static passive radar and how it obtains a target solution. The multi-static passive TDOA radar example in this entry consists of a single receiving station that monitors the direct signals from the transmitters and indirect, multi-path signals from potential targets. Other configurations are possible but for the 2D operation described requires 3 transmitter – receiver combinations. To obtain a target the system uses the time difference of arrival (TDOA) of the multi-path signal is compared to the direct path signal from the same transmitter. Using these timings, solutions for possible target locations can be generated. I will cover the basic bi-static geometry on which this system relies and how it is incorporated into multi-static system. A 2D solution will also be derived from scratch for a multi-static system.

Bi-Static Radar Geometry

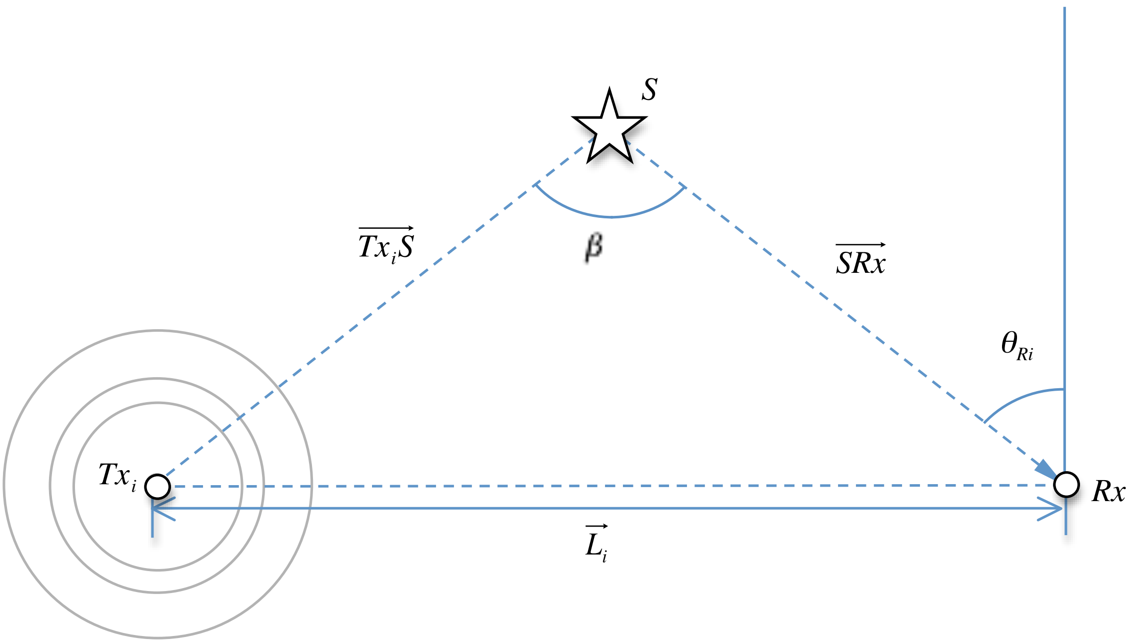

Figure 1

Figure 1 illustrates a simple bi-static radar system consisting of a transmitter Tx_i, a target S and a receiver Rx. A distance separates the transmitter and receiver $|L_i|$. The sum of the distances between the target S and the transmitter $|\overrightarrow{Tx_i S}|$ and the distance between the target and receiver $|\overrightarrow{SRx}|$ is the range of the bi-static radar. By measuring the time difference of arrival t of the reflected path via the target relative to the direct path signal from the transmitter the relationship between the direct path $\overrightarrow{L_i}$ and the bi-static range T_i can be determined as demonstrated in Equation 1

Equation 1

$$T_i = \overrightarrow{L_i} + t_i c\,,$$

where c is the speed of propagation of the signal through air.

Knowing the bi-static range allows the target to be positioned somewhere on an ellipse. The sum of the distances from the points of the ellipse to the transmitter and receiver is equal to the bi-static range T_i.

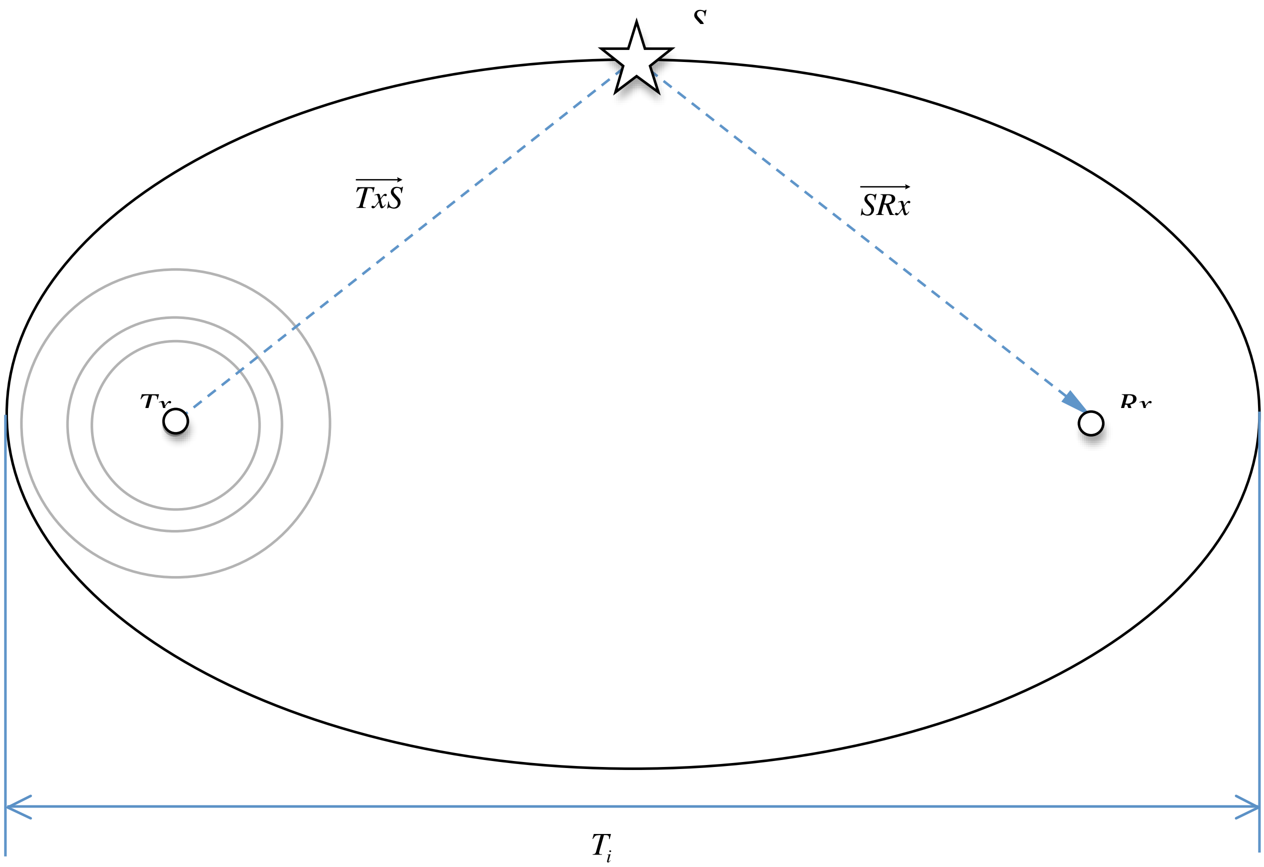

Figure 2

This relationship can also be demonstrated as in Equation 2

Equation 2

$$T_i = \left|\overrightarrow{Tx_i S}\right| + \left|\overrightarrow{SRx}\right|.$$

The location of the transmitter Tx_i is at (x_{ti}, y_{tx}, z_{ti}). The location of the receiver Rx is at (x_r, y_r, z_r) and the target is located (x, y, z). Expanding Equation 2 gives

Equation 3

$$T_i = \sqrt{(x - x_{ti})^2 + (y - y_{ti})^2 + (z - z_{ti})^2} + \sqrt{(x_r - x)^2 + (y_r - y)^2 + (z_r - z)^2}.$$

TDOA Multi-Static Radar Geometry

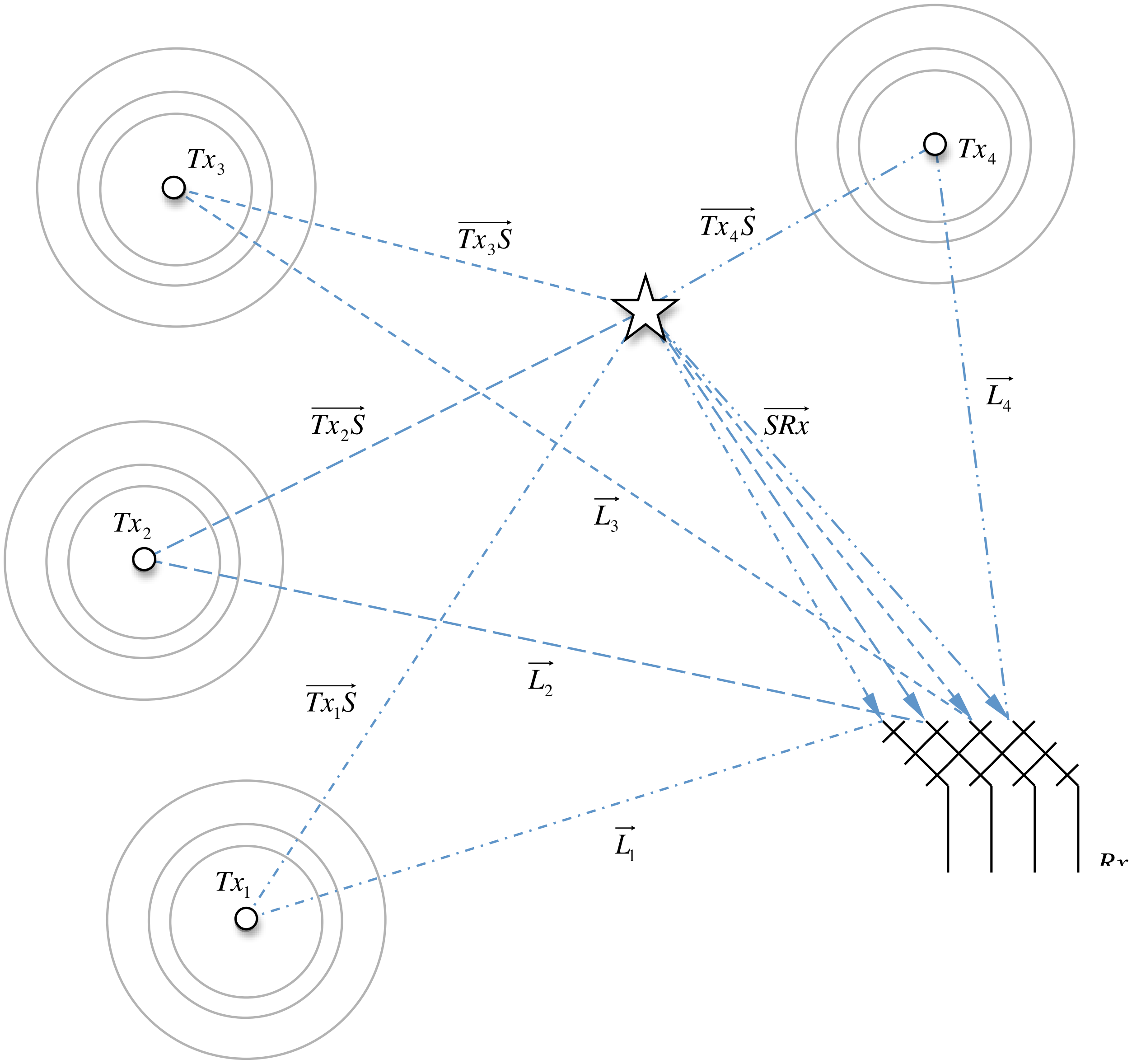

Figure 3

Figure 3 shows a multi-static system consisting of four transmitters, a target and a receiver. There is a direct path signal, L_i from every transmitter to the receiver. A multi path signal from each of the transmitter $\overrightarrow{Tx_iS}$ is reflected by the target S and is intercepted by the receiver. The path the reflected signal follows from the target to the receiver is common for all transmitters $\overrightarrow{SRx}$.

2D Operation

A two dimensional solution for the target can be found using measurements of the TDOA of the reflected signal compared to the direct signal from each transmitter. The

2D solution requires three TDOA measurements from three transmitters T_1, T_2 and T_3 . Using the TDOA values and the known locations of the transmitters Equation 3 can be rewritten for each transmitter

Equation 4

$$T_1 = \sqrt{(x - x_{t1})^2 + (y - y_{t1})^2} + \sqrt{(x_r - x)^2 + (y_r - y)^2}$$

Equation 5

$$T_2 = \sqrt{(x - x_{t2})^2 + (y - y_{t2})^2} + \sqrt{(x_r - x)^2 + (y_r - y)^2}$$

Equation 6

$$T_3 = \sqrt{(x - x_{t3})^2 + (y - y_{t3})^2} + \sqrt{(x_r - x)^2 + (y_r - y)^2}.$$

Equation 4 and Equation 5 can be rearranged into

Equation 7

$$\sqrt{(x - x_{t1})^2 + (y - y_{t1})^2} = T_1 - \sqrt{(x_r - x)^2 + (y_r - y)^2}$$

and

Equation 8

$$\sqrt{(x - x_{t2})^2 + (y - y_{t2})^2} = T_2 - \sqrt{(x_r - x)^2 + (y_r - y)^2}.$$

Squaring both sides of Equation 7 and Equation 8 gives

Equation 9

$$(x - x_{t1})^2 + (y - y_{t1})^2 = T_1^2 - 2T_1\sqrt{(x_r - x)^2 + (y_r - y)^2} + (x_r - x)^2 + (y_r - y)^2$$

Equation 10

$$(x - x_{t2})^2 + (y - y_{t2})^2 = T_2^2 - 2T_2\sqrt{(x_r - x)^2 + (y_r - y)^2} + (x_r - x)^2 + (y_r - y)^2.$$

Rearranging Equation 9 and Equation 10 in terms of $\left|\overrightarrow{SRx}\right|$

Equation 11

$$\sqrt{(x_r - x)^2 + (y_r - y)^2} = \frac{1}{2T_1}\left[T_1^2 + (x_r - x)^2 + (y_r - y)^2 - (x - x_{t1})^2 - (y - y_{t1})^2\right]$$
$$= \frac{1}{2T_1}\left[T_1^2 + x_r^2 - 2x_r x + y_r^2 - 2y_r y + z_r^2 - 2z_r z - x_{t1}^2 + 2x_{t1}x - y_{t1}^2 + 2y_{t1}y - z_{t1}^2 + 2z_{t1}z\right]$$

Equation 12

$$\sqrt{(x_r - x)^2 + (y_r - y)^2} = \frac{1}{2T_2}\left[T_2^2 + (x_r - x)^2 + (y_r - y)^2 - (x - x_{t2})^2 - (y - y_{t2})^2\right]$$
$$= \frac{1}{2T_2}\left[T_2^2 + x_r^2 - 2x_r x + y_r^2 - 2y_r y + z_r^2 - 2z_r z - x_{t2}^2 + 2x_{t2}x - y_{t2}^2 + 2y_{t2}y - z_{t2}^2 + 2z_{t2}z\right]$$

.

Equating Equation 11 and Equation 12 gives

Equation 13

$$\frac{1}{2T_1}\left[T_1^2 + x_r^2 - 2x_r x + y_r^2 - 2y_r y + z_r^2 - 2z_r z - x_{t1}^2 + 2x_{t1}x - y_{t1}^2 + 2y_{t1}y - z_{t1}^2 + 2z_{t1}z\right] =$$
$$\frac{1}{2T_2}\left[T_2^2 + x_r^2 - 2x_r x + y_r^2 - 2y_r y + z_r^2 - 2z_r z - x_{t2}^2 + 2x_{t2}x - y_{t2}^2 + 2y_{t2}y - z_{t2}^2 + 2z_{t2}z\right]$$
$$T_2\left[T_1^2 + x_r^2 - 2x_r x + y_r^2 - 2y_r y + z_r^2 - 2z_r z - x_{t1}^2 + 2x_{t1}x - y_{t1}^2 + 2y_{t1}y - z_{t1}^2 + 2z_{t1}z\right] =$$
$$T_1\left[T_2^2 + x_r^2 - 2x_r x + y_r^2 - 2y_r y + z_r^2 - 2z_r z - x_{t2}^2 + 2x_{t2}x - y_{t2}^2 + 2y_{t2}y - z_{t2}^2 + 2z_{t2}z\right]$$

Rearranging Equation 13Equation 13in the form of

Equation 14

$$y = Ax + B$$

gives

Equation 15

$$\left[2T_2(y_{t1}-y_r)-2T_1(y_{t2}-y_r)\right]y =$$
$$\left[2T_2(x_{t1}-x_r)-2T_1(x_{t2}-x_r)\right]x + T_2\left[T_1^2+x_r^2+y_r^2-x_{t1}^2-y_{t1}^2\right]-T_1\left[T_2^2+x_r^2+y_r^2-x_{t2}^2-y_{t2}^2\right]$$
$$y=\frac{\left[2T_2(x_{t1}-x_r)-2T_1(x_{t2}-x_r)\right]}{\left[2T_2(y_{t1}-y_r)-2T_1(y_{t2}-y_r)\right]}x+\frac{T_2\left[T_1^2+x_r^2+y_r^2-x_{t1}^2-y_{t1}^2\right]-T_1\left[T_2^2+x_r^2+y_r^2-x_{t2}^2-y_{t2}^2\right]}{\left[2T_2(y_{t1}-y_r)-2T_1(y_{t2}-y_r)\right]}$$

Equation 16

$$A=\frac{\left[2T_2(x_{t1}-x_r)-2T_1(x_{t2}-x_r)\right]}{\left[2T_2(y_{t1}-y_r)-2T_1(y_{t2}-y_r)\right]}$$

Equation 17

$$B=\frac{T_2\left[T_1^2+x_r^2+y_r^2-x_{t1}^2-y_{t1}^2\right]-T_1\left[T_2^2+x_r^2+y_r^2-x_{t2}^2-y_{t2}^2\right]}{\left[2T_2(y_{t1}-y_r)-2T_1(y_{t2}-y_r)\right]}$$

Using Equation 3 and Equation 5 rearranging into the form

Equation 18

$$y = Cx + D$$

gives

Equation 19

$$\left[2T_3(y_{t1}-y_r)-2T_1(y_{t3}-y_r)\right]y =$$
$$\left[2T_3(x_{t1}-x_r)-2T_1(x_{t3}-x_r)\right]x + T_3\left[T_1^2+x_r^2+y_r^2-x_{t1}^2-y_{t1}^2\right]-T_1\left[T_3^2+x_r^2+y_r^2-x_{t3}^2-y_{t3}^2\right]$$
$$y=\frac{\left[2T_3(x_{t1}-x_r)-2T_1(x_{t3}-x_r)\right]}{\left[2T_3(y_{t1}-y_r)-2T_1(y_{t3}-y_r)\right]}x+\frac{T_3\left[T_1^2+x_r^2+y_r^2-x_{t1}^2-y_{t1}^2\right]-T_1\left[T_3^2+x_r^2+y_r^2-x_{t3}^2-y_{t3}^2\right]}{\left[2T_3(y_{t1}-y_r)-2T_1(y_{t3}-y_r)\right]}$$

Equation 20

$$C=\frac{\left[2T_3(x_{t1}-x_r)-2T_1(x_{t3}-x_r)\right]}{\left[2T_3(y_{t1}-y_r)-2T_1(y_{t3}-y_r)\right]}$$

Equation 21

$$D=\frac{T_3\left[T_1^2+x_r^2+y_r^2-x_{t1}^2-y_{t1}^2\right]-T_1\left[T_3^2+x_r^2+y_r^2-x_{t3}^2-y_{t3}^2\right]}{\left[2T_3(y_{t1}-y_r)-2T_1(y_{t3}-y_r)\right]}$$

Equating Equation 15 and Equation 19

Equation 22

$$Ax + B = Cx + D$$

and rearranging for x gives

Equation 23

$$x = \frac{D - B}{A - C}.$$

Subtitling x into Equation 22 for y gives

Equation 24

$$y = -\left[A\left(\frac{D - B}{A - C}\right) + B\right]$$

Contact Me

I would appreciate any feedback on my work, positive or negative. I would be especially interested to hear for people in industry or academia as I am currently looking for an opportunity in engineering. I am particularly interested in digital signal processing, FPGAs, algorithm design, MATLAB and system design. By far the easiest way to contact me is by e-mail andrew@chiprate.co.uk. A PGP public key for this address can be found at www.chiprate.co.uk in the contact me section.